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ONE-DIMENSIONAL UNSTEADY MOTIONS OF GAS DISPLACED BY A PISTON

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We consider the flow of a gas displaced by a piston which at some instant begins to expand according to a power law with an exponent smaller than that corresponding to an intense explosion. We assume that the gas has received a finite energy prior to the beginning of motion of the piston. The energy of the gas in this case remains finite over an infinite time interval, so that all of the required functions are obtainable by linearization relative to the values occurring in the problem of an intense explosion. The solution is constructed by investigating the inverse problem in which a shock wave moves through a quiescent gas of constant density and at a pressure negligibly small as compared with the pressure behind it is specified. The piston expansion law is obtained by solving the resulting Cauchy problem. Special attention is given to the case of a cylindrical piston of constant radius, when the required solution contains logarithmic terms.

The problem of motion of gas due to the expansion of a piston at a constant rate was solved by Sedov [1] and Taylor [2]. The more general case in which the velocity of the piston depends on time according to a power law was later investigated by Krasheninnikova [3] and by Kochina and Mel'nikova [4]. In these studies the functions describing the perturbed flow fields depend on the self-similar variable only and are found by integrating a system of nonlinear ordinary differential equations. As may be seen from qualitative investigation [3 and 4], the problem does not always have a solution if the piston motion is defined as $R = ct^n$ (where R is the coordinate and t is the time). In order for a solution to exist, the exponent n must satisfy the condition $n > 2 / (\nu + 2)$, where the parameter $\nu = 1, 2, 3$ for flows with plane, axial, and central symmetry, respectively.

Grigorian [5] proposed a simple explanation of this requirement: according to him, when $n < 2/(\nu + 2)$, the gas immediately receives an infinite amount of energy at the initial instant of piston motion. On the other hand, when $n > 2/(\nu + 2)$, the amount of energy conveyed to the gas at $t = 0$ is equal to zero, and the energy becomes infinite only as $t \to \infty$. If $n = 2/(\nu + 2)$, the finite energy is released over a negligibly short time, i.e. the problem is that of an intense explosion as investigated in detail by Sedov [6 and 7] and Taylor [8].

We shall assume that the piston moves according to a power law beginning at some instant $t = T > 0$, and that $n < 2/(\nu + 2)$. The energy E_T conveyed to the gas in the initial phase of expansion (when $t < T$) is assumed to be finite and not equal to zero. In our case its value remains finite even after infinite time following the initial instant of piston motion. Hence, all of the functions in the required solution can be obtained by linearization relative to the values which they assume in the intense explosion problem. Understandably, linearization is legitimate only if $(E_\infty - E_T)/E_\infty \ll 1$, where E_∞ denotes the total energy in the zone of perturbed gas flow as $t \to \infty$.

1. We denote by r the distance from some point in the space to the plane, axis, or center of symmetry; v is the velocity; ρ is the density; p is the pressure; $\varkappa$ is the ratio of specific heats. The equations of one-dimensional unsteady motions of a cas can be written as [9]

$$\begin{gathered}\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \\ \frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\left[\frac{\partial v}{\partial r} + (\nu - 1)\frac{v}{r}\right] = 0 \\ \frac{\partial p}{\partial t} + v\frac{\partial p}{\partial r} + \varkappa p\left[\frac{\partial v}{\partial r} + (\nu - 1)\frac{v}{r}\right] = 0\end{gathered} \tag{1.1}$$

The density ρ_1 of an unperturbed medium will be assumed constant. As regards the intensity of the shock wave produced by the expanding piston, we assume that it is extremely high. This enables us to neglect the pressure p_1 in front of the discontinuity surface. This means that on passage through the shock front moving at the velocity $c(t)$ we have [9]

$$v_2 = \frac{2}{\varkappa + 1}c, \qquad \rho_2 = \frac{\varkappa + 1}{\varkappa - 1}\rho_1, \qquad p_2 = \frac{2}{\varkappa + 1}\rho_1 c^2 \tag{1.2}$$

where the subscript 2 refers to the gas in the compressed state. In constructing the solution of system of differential equations (1.1) we must require fulfillment not only of Hugoniot conditions (1.2), but also of a boundary condition whereby the gas particles adjacent to the piston propagate with the same velocity as the latter.

Instead of solving the above problem it is convenient to invert the procedure and to specify the coordinate $r_s(t)$ of the shock wave front for $t > T$ as follows:

$$r_s(t) = (at)^{\frac{2}{\nu+2}}\left(1 - \varepsilon t^{-\frac{2m}{\nu+2}}\right)$$

Here ε is a small parameter, and the constant a is determined by the energy E_∞ released in the initial phase of expansion of the gas for $t < T$ and throughout the entire period of piston motion; the exponent $m > 0$. We assume that the initial energy E_T differs only slightly from the total energy of perturbed flow, i.e. that $(E_\infty - E_T)/E_\infty \ll 1$. Essentially, this assumption is already embodied in our law of propagation of the shock front. Converting to the new independent variables t and $\lambda = r(at)^{-2/(\nu+2)}$, we write the required functions as

$$v = v_{20}\left[f(\lambda) + \varepsilon t^{-\frac{2m}{\nu+2}} f_m(\lambda)\right], \quad \rho = \rho_{20}\left[g(\lambda) + \varepsilon t^{-\frac{2m}{\nu+2}} g_m(\lambda)\right]$$

$$p = p_{20}\left[h(\lambda) + \varepsilon t^{-\frac{2m}{\nu+2}} h_m(\lambda)\right] \qquad \left(\rho_{20}(t) = \frac{(\varkappa+1)\rho_1}{\varkappa-1}\right) \tag{1.3}$$

The quantitiies $v_{20}(t)$ and $p_{20}(t)$ in Formulas (1.3) are given by conditions (1.2) if we assume that

$$c = \frac{2}{\nu+2} a^{\frac{2}{\nu+2}} t^{-\frac{\nu}{\nu+2}}$$

As regards the functions $f(\lambda)$, $g(\lambda)$ and $h(\lambda)$, they are clearly the solution of the intense explosion problem [9]. In accordance with expansion (1.3) we can represent the equation of the shock wave front in the form

$$\lambda_s = 1 - \varepsilon t^{-\frac{2m}{\nu+2}} \tag{1.4}$$

where by standard procedure of the method of perturbations the boundary conditions for the required functions $f_m(\lambda)$, $g_m(\lambda)$ and $h_m(\lambda)$ must refer to the point $\lambda = 1$. Retaining only first-order terms in ε in all the relations and neglecting the terms of higher orders of smallness, for $\lambda = 1$ we obtain

$$f_m = m - 1 + \frac{df}{d\lambda}, \qquad g_m = \frac{dg}{d\lambda}, \qquad h_m = 2(m-1) + \frac{dh}{d\lambda} \tag{1.5}$$

Linearization of equations of motion (1.1) can be effected in precisely similar fashion. This yields the homogeneous system

$$\begin{gathered}
\left(f - \frac{\varkappa+1}{2}\lambda\right) g \frac{df_m}{d\lambda} + \frac{\varkappa-1}{2}\frac{dh_m}{d\lambda} + \left[\frac{df}{d\lambda} - \frac{(\nu+2m)(\varkappa+1)}{4}\right] g f_m + \\
+ \left[\left(f - \frac{\varkappa+1}{2}\lambda\right)\frac{df}{d\lambda} - \frac{\nu(\varkappa+1)}{4} f\right] g_m = 0 \\[2ex]
g\frac{df_m}{d\lambda} + \left(f - \frac{\varkappa+1}{2}\lambda\right)\frac{dg_m}{d\lambda} + \left(\frac{dg}{d\lambda} + \frac{\nu-1}{\lambda} g\right) f_m + \\
+ \left(\frac{df}{d\lambda} + \frac{\nu-1}{\lambda} f - \frac{\varkappa+1}{2} m\right) g_m = 0 \\[2ex]
\varkappa h \frac{df_m}{d\lambda} + \left(f - \frac{\varkappa+1}{2}\lambda\right)\frac{dh_m}{d\lambda} + \left[\frac{dh}{d\lambda} + \frac{\varkappa(\nu-1)}{\lambda} h\right] f_m + \\
+ \left[\varkappa\frac{df}{d\lambda} + \frac{\varkappa(\nu-1)}{\lambda} f - \frac{(\varkappa+1)(\nu+m)}{2}\right] h_m = 0
\end{gathered} \tag{1.6}$$

which gives us the functions $f_m(\lambda)$, $g_m(\lambda)$ and $h_m(\lambda)$ on the segment $0 < \lambda < 1$.

We can immediately point out several exact solutions of Cauchy problem (1.5) for Eqs. (1.6) on the basis of the group properties of the problem of an intense point explosion. As we know, its self-similarity is associated with the existence of a certain group of similarity transformations. The shift in the energy E_∞ again yields the solution of the intense explosion problem with a somewhat altered value of this parameter. Moreover, the initial Euler equations and the Hugoniot conditions are invariant relative to the shift in the time t, and for $\nu = 1$ in the coordinate r as well.

Bearing in mind the above remarks, we can write

$$f_m = -f + \lambda\frac{df}{d\lambda}, \quad g_m = \lambda\frac{dg}{d\lambda}, \quad h_m = -2h + \lambda\frac{dh}{d\lambda} \quad \text{for } m = 0 \tag{1.7}$$

$$f_m = \frac{\nu}{2} f + \lambda\frac{df}{d\lambda}, \quad g_m = \lambda\frac{dg}{d\lambda}, \quad h_m = \nu h + \lambda\frac{dh}{d\lambda} \quad \text{for } m = \frac{\nu+2}{2}$$

$$f_m = \frac{df}{d\lambda}, \qquad g_m = \frac{dg}{d\lambda}, \qquad h_m = \frac{dh}{d\lambda} \qquad \text{for } m = \nu = 1 \tag{1.7}$$
(cont.)

Altough the first of Formulas (1.7) is valid for all ν, it is not explicitly dependent on this parameter. This procedure for finding particular solutions of the linearized system of equations which relies essentially on the invariance of the fundamental solution relative to some transformation group was first used by Zel'dovich and Barenblatt in the theory of unsteady gas filtration [10].

2. The inverse problem under consideration enables us to associate each shock wave propagation law (1.4) with a quite specific law of piston motion. In order to investigate its character we must find the asymptotic behavior of the function $f_m(\lambda)$ as $\lambda \to 0$. It is convenient to begin by transforming system of Eqs. (1.6) to a single third-order equation for $f_m(\lambda)$ and then to obtain the asymptotic expansions of all its linearly independent particular solutions as $\lambda \to 0$. In order to abbreviate our formulations we make use of the first integral of system (1.6). The existence of such an integral for the equations in variations taken relative to the self-similar solutions describing unsteady one-dimensional flows was proved in [11 and 12]. Using the methods described there, we immediately obtain

$$\frac{2\nu f_m}{2f - (\varkappa + 1)\lambda} + [m\varkappa - \nu(\varkappa - 1)]\frac{g_m}{g} + (\nu - m)\frac{h_m}{h} = \nu C\left(\frac{h}{g^\varkappa}\right)^{\frac{m}{\nu}} \tag{2.1}$$

The constant C occurring in this expression can be determined by Cauchy data (1.5). Making use of Formula (2.1), we can transform system (1.6) to a second-order rather than a third-order equation for the formula $f_m(\lambda)$; this equation is inhomogeneous. The asymptotic expansions of its solutions as $\lambda \to 0$ can be found readily, after which the asymptotic expansions of the functions $g_m(\lambda)$ and $h_m(\lambda)$ are easy to obtain. As a result, for the first linearly independent solution of initial system (1.6) we have

$$f_m = \alpha_1\lambda + \ldots, \qquad g_m = \alpha_2\lambda^{\nu/(\varkappa-1)} + \ldots, \qquad h_m = \alpha_3 + \ldots \tag{2.2}$$

We take the second of the required solutions in the form

$$f_m = \beta_1\lambda^{1-\nu} + \ldots, \qquad g_m = \beta_2\lambda^{\nu(2-\varkappa)/(\varkappa-1)} + \ldots, \qquad h_m = \beta_3\lambda^{\nu(2-\varkappa)/(\varkappa-1)+2} + \ldots \tag{2.3}$$

We write the third solution of system (1.6) as

$$f_m = \gamma_1\lambda^{(\nu-m\varkappa)/(\varkappa-1)+2} + \ldots, \qquad g_m = \gamma_2\lambda^{(\nu-m\varkappa)/(\varkappa-1)} + \ldots$$

$$h_m = \gamma_3\lambda^{(\nu-m\varkappa)/(\varkappa-1)+2}. \tag{2.4}$$

The constants denoted by the same letters but different subscripts are related in certain ways. We shall not write out these relationships because of their cumbersomness.

Now let us find the pison coordinate $R = \delta r_m(t)$. The gas particles adjacent to its surface must propagate with the same velocity as the piston. This condition yields

$$\delta\frac{dr_m}{dt} = v_{20}\left[f(\lambda) + \varepsilon t^{-\frac{2m}{\nu+2}} f_m(\lambda)\right]_{r=\delta r_m(t)}$$

Substituting in the first of Formulas (2.2)–(2.4) which are valid for $\lambda \to 0$ and retaining only the principal terms in the resulting expression, we obtain the following equation for determining $r_m(t)$:

$$\frac{dr_m^\nu}{dt} - \frac{2\nu}{\varkappa(\nu+2)}\frac{r_m^\nu}{t} = 4\varepsilon\delta^{-\nu}\frac{\nu\beta_1}{(\varkappa+1)(\varkappa+2)}a^{\frac{2\nu}{\nu+2}}t^{-\frac{2(m-\nu)}{\nu+2}-1} \tag{2.5}$$

The solution

$$r_m{}^0 = At^{\frac{2\nu}{\varkappa(\nu+2)}} \tag{2.6}$$

of the corresponding homogeneous equation yields the trajectories of the gas particles in the neighborhood of the plane, axis, or center of symmetry [9]. By suitable choice of the piston coordinate at $t = T$ we can always ensure that the constant A in Formula (2.6) is equal to zero. It is clear from this that the solution of the inhomogeneous equation (2.5) is the required law of piston expansion.

For simplicity we set $|\varepsilon| = \delta^\nu$ and choose the sign of ε in such a way that the functions $r_m(t)$ and $R(t)$ are positive. For $m \neq \nu(\varkappa - 1)/\varkappa$ we have

$$r_m = a^{\frac{2}{\nu+2}} \left| \frac{2\varkappa\nu\beta_1}{(\varkappa+1)(\nu\varkappa - m\varkappa - \nu)} \right|^{\frac{1}{\nu}} t^{\frac{2(\nu-m)}{\nu(\nu+2)}} \tag{2.7}$$

The above relation shows that the law of piston motion is determined only by integral (2.3) of system (1.6) and does not depend on the constants α_1 and γ_1 in its two other particular solutions. As can be shown by numerical integration of Eqs. (1.6), the constant β_1 is positive for $m < \nu$. This condition must be fulfilled in order for the exponent of t in Formula (2.7) to be larger than zero. Hence, for $m < \nu(\varkappa - 1)/\varkappa$ the parameter $\varepsilon > 0$. Under these conditions as $t \to \infty$ the piston moves more rapidly than the particles in an intense explosion. As regards the coordinate of the wave front, its values with increasing time tend to the corresponding values in the intense explosion problem, but always remain smaller than the latter.

For $m > \nu(\varkappa - 1)/\varkappa$ the sign of ε is negative. The velocity of the piston is in this case smaller than the velocity of the particles brought into motion by a violent explosion, and the shock front coordinate reaches its top limit.

If $m = \nu(\varkappa - 1)/\varkappa$ we must replace Formula (2.7) by

$$r_m = a^{\frac{2}{\nu+2}} \left[A + 4\varepsilon\delta^{-\nu} \frac{\nu\varkappa\beta_1}{(\varkappa+1)(\nu\varkappa + 2\varkappa - 2\nu)} \ln t \right]^{\frac{1}{\nu}} t^{\frac{2}{\varkappa(\nu+2)}} \tag{2.8}$$

Here we have retained the term corresponding to the homogeneous solution. For $\varepsilon = 0$ the law of piston motion is determined by the self-similar integral of the violent explosion problem. For $\varepsilon \neq 0$ the difference between the piston velocity and the velocity of propagation of gas particles propelled by an explosion wave turns out to be logatithmic.

Strictly speaking, the adopted form (1.3) of representing the solution of the problem under consideration is not valid for small values of the coordinate r. As we infer from expansions (2.2)–(2.4) and Formula (2.7), in the neighborhood of the piston surface both of the terms appearing in square brackets in the first of two relations of (1.3) are of the same order. In order to eliminate this difficulty and to obtain the solution valid within the approximation under consideration, we can make use of the familiar method of combining the outer and inner asymptotic expansions. The essence of this method is described in detail in Van Dyke's book [13]. Omitting the appropriate expressions, we merely note that the application of the method of combining asymptotic expansions confirms the results of the linear theory, even though these were obtained without allowance for the special character of the solution for the small values of the coordinate r. In particular, laws of piston motion (2.7) and (2.8) remain valid.

To illustrate, let us cite some relations obtained by numerical integration of system (1.6) with allowance for Cauchy data (1.5). In our computations we assumed the ratio of specific heats to be 1.4. As we see from Fig. 1, the function $f_m(\lambda)$ is positive. For $\nu = 1$ its values are finite for all λ, and for $\nu = 2.3$ it tends fairly rapidly to infinity as $\lambda \to 0$. Only in the exceptional cases realized for $m = 0$ and $m = (\nu + 2) / 2$ do we have $f_m(0) = 0$. For the same ν the values of the function $g_m(\lambda)$ are very close to those shown in Fig. 2, although the parameter m is arbitrary. The large values of $g_m(\lambda)$ for λ close to unity and their rapid decrease as the independent variables decreases mean that in perturbed motion the gas is concentrated near the shock wave front. The curves in Fig. 3 represent the pressure variation. For $m = \nu = 1$ the solution of this problem results from the last formulas of (1.7), from which we find, for example, that $\beta_1 = (\varkappa + 1) / 2\varkappa$.

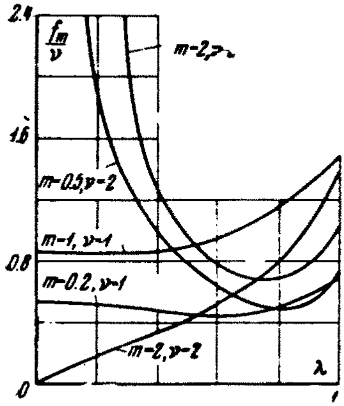

Fig. 1

3. The coefficient β_1 in expansion (2.3) vanishes when $m = (\nu + 2) / 2$. This value of the parameter is proper, since system (1.6) has a nonzero

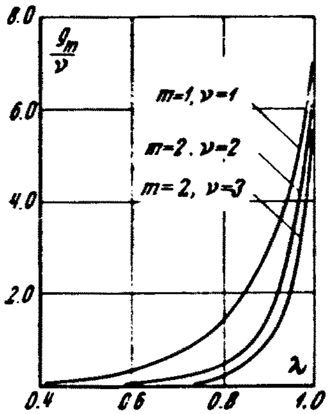

Fig. 2

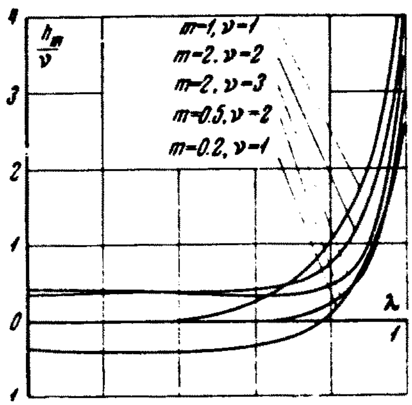

Fig. 3

solution even if as our boundary condition we require that the gas particle velocity vanish on approaching the plane, axis, or center of symmetry. As already noted, for $m = (\nu + 2) / 2$ the solution of system (1.6) can be expressed with the aid of the second group of Formulas (1.7). This representation is related to the invariance of the initial Euler equations and the Hugoniot conditions relative to the time shift. It is clear that expansion (1.3) cannot describe the piston motion for $m = (\nu + 2) / 2$.

Let us invert the procedure as above and specify the coordinate of the shock wave front. Instead of Formula (1.4) we write

$$\lambda_* = 1 - \varepsilon t^{-1}\left(1 + \frac{2}{\nu + 2} b \ln t\right) \tag{3.1}$$

The constant b appearing in this expression is arbitrary. In accordance with (3.1), we can write the expansion of the required solution in the form

$$
\begin{aligned}
v &= v_{20}\left\{f(\lambda) + \varepsilon t^{-1}\left[\frac{2}{\nu+2} b f_m(\lambda)\ln t + f_{m1}(\lambda)\right]\right\} \\
\rho &= \rho_{20}\left\{g(\lambda) + \varepsilon t^{-1}\left[\frac{2}{\nu+2} b g_m(\lambda)\ln t + g_{m1}(\lambda)\right]\right\} \\
p &= p_{20}\left\{h(\lambda) + \varepsilon t^{-1}\left[\frac{2}{\nu+2} b h_m(\lambda)\ln t + h_{m1}(\lambda)\right]\right\}
\end{aligned}
\tag{3.2}
$$

The initial values of all the functions to be determined are obtainable from conditions (1.2) at the shock front. Using the method of variations, we can easily prove that for $\lambda = 1$ the functions $f_m(\lambda)$, $g_m(\lambda)$ and $h_m(\lambda)$ satisfy Eqs. (1.5). For the initial values $f_{m1}(\lambda)$, $g_{m1}(\lambda)$ and $h_{m1}(\lambda)$ of these functions we have the relations

$$
f_{m1}(1) = m - 1 - b + \frac{df}{d\lambda}, \qquad g_{m1}(1) = \frac{dg}{d\lambda}
$$
$$
h_{m1}(1) = 2(m - 1 - b) + \frac{dh}{d\lambda} \tag{3.3}
$$

Substitution of Formulas (3.2) into Euler equations (1.1) shows that not only the initial data, but also the system of ordinary differential equations which determines the functions $f_m(\lambda)$, $g_m(\lambda)$ and $h_m(\lambda)$ on the segment $0 < \lambda < 1$ remain constant. Thus, we can represent them, as before, with the aid of the second formulas of (1.7). As regards the functions $f_{m1}(\lambda)$, $g_{m1}(\lambda)$ and $h_{m1}(\lambda)$, they satisfy a system of inhomogeneous differential equations whose left sides coincide with (1.6) and whose right sides are the quantities

$$
-\frac{\varkappa+1}{2} bg\left(\frac{\nu}{2} f + \lambda \frac{df}{d\lambda}\right), \quad -\frac{\varkappa+1}{2} b\lambda \frac{dg}{d\lambda}, \quad -\frac{\varkappa+1}{2} b\left(\nu h + \lambda \frac{dh}{d\lambda}\right) \tag{3.4}
$$

Investigation of the asymptotic behavior of the particular solutions of the above inhomogeneous equations proves the possibility of using them to determine the expansion of the piston. The law of motion of the latter corresponds to (2.7). In other words,

$$
r_m \sim t^{(\nu-2)/\nu(\nu+2)} \tag{3.5}
$$

To obtain the values of the required functions by numerical integration of the equations it is convenient to write them in the form of sums,

$$
f_{m1} = f_m + bf_m^*, \qquad g_{m1} = g_m + bg_m^*, \qquad h_{m1} = h_m + bh_m^* \tag{3.6}
$$

Here the quantities $f_m^*(\lambda)$, $g_m^*(\lambda)$ and $h_m^*(\lambda)$ satisfy system (1.6) with right sides (3.4) in which the arbitrary constant $b = 1$. The initial data for the new functions can be obtained by substituting Eqs. (3.5) into (3.3) and turn out to be

$$
f_m^*(1) = -1, \qquad g_m^*(1) = 0, \qquad h_m^*(1) = -2
$$

The results of integration for $m = \nu = 2$ appear in Fig. 4.

For $(\nu + 2)/2 < m < \nu$ the solution of the problem of a piston expanding in a quiescent gas of constant density at negligibly low pressure should be taken as a sum of three terms. The first of these terms is associated with the self-similar solution [9] describing the propagation of blast waves; the second is associated with solution (1.7) for $m = (\nu + 2)/2$. Only the third term is directly related to the motion of the piston according to law (2.7). If we assume that the exponent of t in this formula is larger than zero, then a solution in the form of a three-term sum must be used only for describing flows with central symmetry.

4. Let us consider the problem of the energy in the zone of perturbed flow confined between the shock wave and piston. At some instant t we have

$$E = e_\nu \int\limits_{R(t)}^{r_*(t)} \left(\frac{\rho v^2}{2} + \frac{p}{\varkappa - 1}\right) r^{\nu-1}\, dr$$

$$e_\nu = 2(\nu - 1)\pi + (\nu - 2)(\nu - 3) \qquad (4.1)$$

Substitution of relations (1.3), (1.4) and (2.7) into Eq. (4.1) yields

$$E = \frac{8a^2 e_\nu}{(\nu + 2)^2 (\varkappa^2 - 1)} \left(J + \varepsilon J_m t^{-\frac{2m}{\nu+2}}\right) \qquad (4.2)$$

Here the constant J is determined by the solution of the intense explosion problem [9] and is proportional to the asymptotic value E_∞ of the gas energy as $t \to \infty$; the value of J_m for $m \neq \nu(\varkappa - 1)/\varkappa$ is given by Formula

$$J_m = \int\limits_0^1 (f^2 g_m + 2fgf_m + h_m)\lambda^{\nu-1}\, d\lambda - 2 - \frac{2\nu\varkappa}{(\varkappa + 1)(\nu\varkappa - m\varkappa - \nu)} h(0) \lim_{\lambda \to 0} (\lambda^{\nu-1} f_m)$$

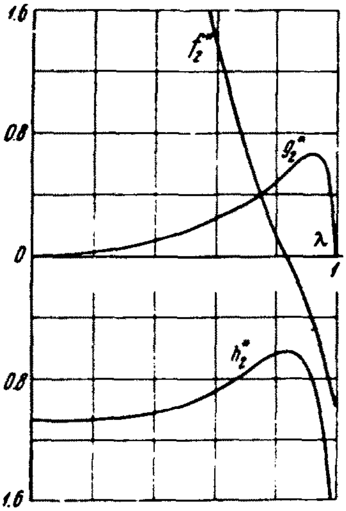

Fig. 4

For $m = \nu(\varkappa - 1)/\varkappa$ the expression for J_m should be replaced in accordance with (2.8) by

$$J_m = \int\limits_0^1 (f^2 g_m + 2fgf_m + h_m)\lambda^{\nu-1}\, d\lambda - 2 - \left[A\varepsilon^{-1}\delta^\nu + 4\nu\varkappa\beta_1 \frac{\ln t}{(\varkappa - 1)(\nu\varkappa + 2\varkappa - 2\nu)}\right]$$

Computations show that the prpduct εJ_m is negative for all $m < \nu$. For $m = \nu$ the exponent of t in the law of piston motion (2.7) vanishes. In this case the piston reaches its maximum dimensions in a finite time $t \leqslant T$ and then stops. For $t > T$ it clearly does no work. In full agreement with this fact, the quantity J_m vanishes for $m = \nu$. It is easy to show that an expression of the form (4.2) for computing the total gas energy is valid if $m = (\nu + 2)/2$; the term proportional to $J_{m_1} t^{-2m/(\nu+2)} \ln t$ vanishes, since $J_{m1} = 0$.

In conclusion we note that by virtue of the familiar analogy [14 and 15] between hypersonic flow past slender bodies and unsteady flows in a space with one less dimension, the solutions obtained above can be used to compute the parameters of the gas between the head shock wave and the surface of a body. The condition which makes this application legitimate clearly reduces to the smallness of the drag at the side of the surface as compared with the drag at the blunt nose. The effect which entails the appearance of a high-entropy layer must be considered separately. The effect of the bluntness of the nose of the body on hypersonic gas flow past it was investigated in this formulation by Chernyi [16–18], Cheng and Pallone [19], and Lees and Kubota [20]. In accordance with the above analogy, Formulas (3.2) also describe flow past a blunt-nosed circular cylinder of finite thickness pointing into the oncoming stream, in such a way that its generatirices are parallel to the unperturbed velocity vector. This statement follows directly from relation (3.5) if we set $\nu = 2$.

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